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Creep of polymer foams

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The linear and non-linear creep of polymeric foams are analysed. Measurements of the creep of rigid polyurethane foams under constant shear indicate that their behaviour is linear at stresses up to half the shear strength; the model describes the data well.

1. Introduction

There is growing interest in the use of polymer foam core sandwich panels as load-bearing components in buildings. In addition to offering a high stiffness per unit weight and excellent thermal insulation, such panels may be easily mass produced. Polymers creep at room temperature, however, limiting their use in structural applications. In order to exploit fully the potential of polymer foam core structural sandwich panels there is a need to understand their creep behaviour. The present work aimed to develop models for the creep of rigid polymer foams in terms of the creep of the solid polymer and the initial moduli of the foam and the solid.

At the simplest level, many solid polymers are linear viscoelastic: that is, creep strains at a given time vary linearly with stress under a constant load. Linear viscoelasticity can be described in a phenomenological way using spring dashpot models or in a mechanistic way using transition state theories [1]. At large strains, or long loading times, many polymers become non-linear viscoelastic: the strain at a given time is no longer proportional to the applied stress. Descriptions of non-linear viscoelasticity rely on empirical functions as the mechanism of non-linear viscoelasticity is not yet well understood.

Although there have been several experimental studies of the creep of polymer foams, little modelling has been done. The following section provides a review of the available literature. Then models for the creep of both linear and non-linear viscoelastic polymeric foams, based on the model of Gibson and Ashby [2, 3], are developed. The results are compared with data from 1200 h creep shear tests on several densities of rigid polyurethane foam; agreement is good.

2. Literature review

2.1. Creep of solid polymers

The creep of a foam depends on that of the solid from which it is made. At the simplest level, polymers may be linear viscoelastic, such that in a creep test the strain at any given time is proportional to the applied stress; the constant of proportionality is given by the time-dependent creep compliance. Linear viscoelastic polymers obey the Boltzman superposition principle: the strain resulting from a complex loading history can be obtained by simply summing the strains for each load increment [1].

If loaded to large strains (or stresses) or for long periods of time, many polymers become non-linear viscoelastic: the strain at a given time is no longer linearly proportional to stress. Their behaviour is described either by empirical equations or by general rheological constitutive equations, neither of which, however, gives any insight into the underlying mechanisms of deformation; this awaits further development. Empirical equations for non-linear viscoelasticity include those of Pao and Marin [4, 5], Findley and Khosla [6] and Van Holde [7]. Pao and Marin have proposed a power law dependence of strain, ε, on stress, σ, similar to that describing the creep of metals

$$\varepsilon(\sigma, t) = \sigma/E + K\sigma^n(1 - e^{-qt}) + B\sigma^n t \quad (1)$$

where E, K, n, q and B are material constants and t is time. Findley and Khosla [6] used a power law time dependence in conjunction with a hyperbolic sine stress dependence:

$$\varepsilon(t) = \varepsilon_0 + mt^n \quad (2)$$

$$\varepsilon(\sigma, t) = \varepsilon'_0 \sinh\frac{\sigma}{\sigma_e} + m't^n \sinh\frac{\sigma}{\sigma_m} \quad (3)$$

where ε'_0, σ_e, m', n and σ_m are material constants. Van Holde [7] has suggested a variation on this with $\sigma \ll \sigma_e$, $n = 1/3$ and $1/\sigma_m = \alpha$, giving

$$\varepsilon(\sigma, t) = \varepsilon_0 + m't^{1/3}\sinh\alpha\sigma \quad (4)$$

where ε_0, m' and α are again material constants. Note that the last two expressions reduce to Equation 2 if σ/σ_e and σ/σ_m are small or if $\alpha\sigma$ is small. All of these empirical equations are for creep under a single increment of load; none is adequate for the prediction of creep under more complex stress histories or stress states.

For this, more general rheological constitutive equations are theoretically required: Ward and Onat [8] have found that Green and Rivlin's multiple integral representation of non-linear viscoelasticity [9–11] is applicable. The difficulty with this approach is that it is complex, requiring a large number of material parameters in the creep equation. The number depends on the stress state: for the simple case of pure tension it is 7, for combined tension and torsion it is 19. Studies comparing the results of multiple integral representation with those of other, simpler representations such as the hyperbolic sin or non-linear Maxwell models show that multiple integral representation is no better at predicting creep under complex stress histories and stress states [12, 13].

For practical purposes, at least under simple stress states and load histories, we make use of the following empirical equation describing the non-linear viscoelastic behaviour of solid polymers

$$\begin{aligned} \varepsilon_s &= \varepsilon_0 + mt^n \\ &= (\varepsilon'_0 + m't^n)\sinh(\sigma_s/\sigma_0) \end{aligned} \tag{5a}$$

where ε_s and σ_s are the strain and stress in the solid, respectively, and ε'_0, m', n and σ_0 are the creep parameters of the solid. It is equivalent to Equation 3 with $\sigma_e = \sigma_m = \sigma_0$; it is preferred as it retains the hyperbolic sine dependence on stress yet is simpler to analyse. Note that it reduces to the linear viscoelastic equation

$$\varepsilon_s = (\varepsilon'_0 + m't^n)\frac{\sigma_s}{E_s} \tag{5b}$$

for $\sigma_s \ll \sigma_0$ and with $\sigma_0 = E_s$.

2.2. Creep of polymer foams

Several experimental studies of creep in polyurethane and polystyrene foams have been carried out. Findley and Stanley [14] measured the creep response over a 2 h period of a single-density (330 kg m^{-3}) rigid polyurethane foam under uniaxial tension and compression, pure torsion, and combined tension and torsion for incremental stress histories. Stress levels varied from 25% to 100% of the yield strength of the material, as defined by the first deviation from linearity in the stress–strain curve. They found that the material was non-linear and that the multiple integral relationship gave a good description of creep for both multiaxial stress states and for complex stress histories. Because the application of the multiple integral theory is complex, they also used a modified superposition principle to estimate their results; this, too, gave satisfactory results. Following this study, Nolte and Findley [15] compared the creep behaviour of fully dense, solid polyurethane with that of the foamed polyurethane of Findley and Stanley [14]. They found that for stress levels less than about two-thirds of the plastic yield strength, the creep strain of the foam could be found simply by multiplying the creep function of the solid times the ratio of the initial stiffnesses of the solid and foam. As the stress level in the foam increased, this simple approach increasingly underpredicted the measured creep strains. As is discussed below, these results suggest that the rigid polyurethane foam tested was linear at stress levels up to about two-thirds of the yield stress, becoming increasingly non-linear at higher stress levels; the linear data are well described by an equation of the form of Equation 2.

Davies [16] reports the results of Just [17] on creep of sandwich beams with flat metal faces and rigid polyurethane foam cores. The creep of the beams is governed by the creep of the foam core in shear; the results indicate that the creep of the foam varies according to $\gamma = \gamma_0 + mt^n$ for times of up to 10 y; n was found to be 0.36. The results indicated that creep parameters estimated from 1000 h tests gave a good prediction of creep after 10 y.

Brown [18] and Hart *et al.* [19] report the results of creep tests on foamed polystyrene. Brown's data at stress levels below the compressive collapse stress of the foam are too limited to determine their linearity. Hart and his colleagues measured the long-term (1000 to 10 000 h) creep at temperatures between 23 and 50 °C. At temperatures up to 45 °C and at stress levels of 0.28 and 0.41 MPa, the polystyrene foam was linear viscoelastic while at 50 °C it became markedly non-linear (the glass temperature of polystyrene is about 110 °C). They showed that the activation energy for the creep process in the foamed polystyrene was within 5% of that of solid polystyrene.

From the experimental studies on the creep of foamed polymers there is general agreement on the following. The strain, ε, at a constant stress, σ, can be described by

$$\varepsilon = \varepsilon_0 + mt^n \tag{6}$$

At low stresses relative to the yield stress and at low temperatures relative to the glass temperature, rigid polyurethane and polystyrene foams are linear viscoelastic while at high stresses and temperatures they become non-linear viscoelastic.

The only model for creep of foams is that of Gibson and Ashby [3] who describe the secondary, steady state creep strain rate of a foam made from a solid obeying power-law creep; this model is inapplicable to polymer foams as polymers do not develop a steady state creep rate.

3. Modelling

We consider the creep of a foam made from a linear viscoelastic solid; the non-linear viscoelastic case is described in the Appendix. We assume that the solid cell wall material is isotropic and has the same creep parameters in both tension and compression. The derivation is based on dimensional arguments used by Gibson and Ashby [2] to estimate the Young's modulus of an isotropic foam with a linearly elastic cell-wall material. Their argument can be briefly summarized as follows. A cell in an open-cell foam is represented by the cubic structure shown in Fig. 1a; because the analysis is based on dimensional arguments, the exact geometry of the cell is not important. Each cell wall has a length, l, and cross-section, b^2. The important feature here is that the adjacent cell walls are staggered so that a uniaxial load on the foam induces bending in the cell walls, as is observed in foams. A load F on the cell wall (Fig. 1b) produces a bending deflection, δ, proportional to Fl^3/E_sI, where E_s is the Young's modulus of the cell-wall material and I is the second moment of area of the wall's cross-section. The remote stress, σ, is proportional to F/l^2, while the remote strain, ε, is proportional to δ/l. Combining these expressions and noting that the relative density of the

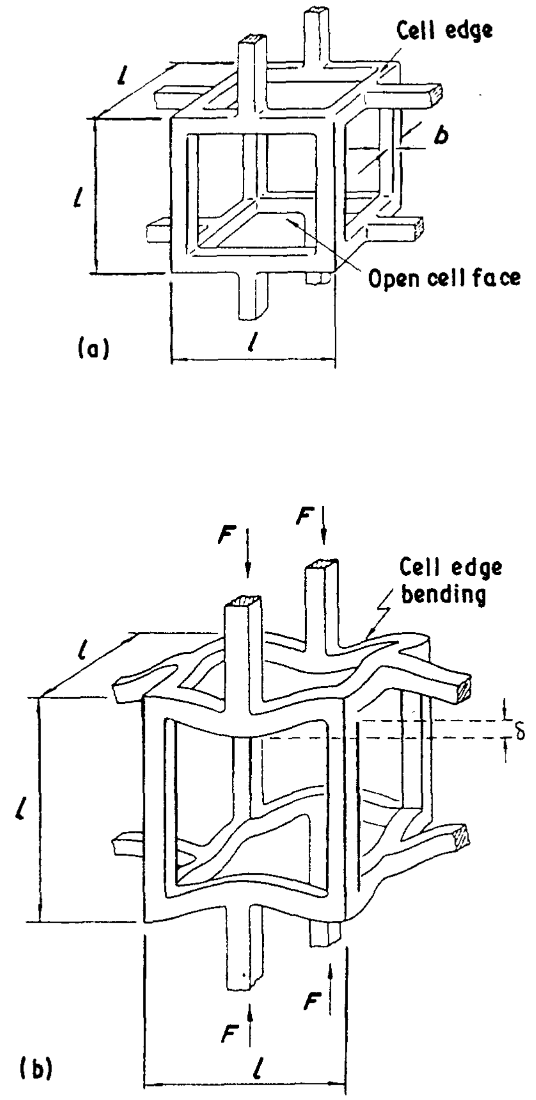

Figure 1 (a) Cubic cell used for dimensional analysis of foam modulus, (b) the deflection, δ, of the cell walls under a load, F.

foam, (ρ^*/ρ_s), is proportional to $(b/l)^2$ gives the Young's modulus of the foam, E^*

$$E^*/E_s = C_1(\rho^*/\rho_s)^2 \tag{7}$$

where C_1 is a constant of proportionality relating to the cell geometry. A similar analysis leads to a relative shear modulus for a foam of

$$G^*/G_s = C_1'(\rho^*/\rho_s)^2 \tag{8}$$

Gibson and Ashby have found that both results give a good description of the moduli of open-cell foams with $C_1 = 1$ and $C_1' = 0.4$. Equations 7 and 8 can also be used for closed-cell foams in which most of solid is concentrated in the cell edges; many polymer foams are like this as a result of surface tension forces which draw material from the faces into the edges during the foaming process.

Now consider the linear viscoelastic behaviour of a foam in which the solid cell-wall material creeps according to Equation 5b

$$\varepsilon_s = (\varepsilon_0' + m't^n)\frac{\sigma_s}{E_s} \tag{9}$$

where E_s is the Young's modulus of the solid at zero time. Note that if this equation is used to describe the long-term creep rather than the short-term creep of the solid, ε_0 may not be equal to one. A stress, σ, acting on the foam induces bending moments in the cell wall, M, proportional to σl^3. The bending moments, in turn, induce internal stresses within the solid cell wall material, σ_s, which are proportional to M/b^3. Combining these expressions gives $\sigma_s \propto \sigma(l/b)^3$. The bending moments also produce deflections, δ, in the cell wall. At zero time, this is

$$\delta \propto \frac{Ml^2}{E_s I} \tag{10}$$

We can also write that the curvature of the bent walls at zero time, κ, is proportional to the strain in the cell wall, ε_s

$$\kappa \propto \frac{\varepsilon_s}{b} \propto \frac{M}{E_s I} \tag{11}$$

where, b, as before, is the thickness of the cell wall.

Combining the last two expressions we obtain

$$\varepsilon_s \propto \frac{\delta b}{l^2} \tag{12}$$

And because δ/l is simply the strain in the bulk foam, ε

$$\varepsilon \propto \varepsilon_s \frac{l}{b} \propto (\varepsilon_0' + m't^n)\frac{\sigma_s}{E_s}\left(\frac{l}{b}\right) \propto (\varepsilon_0' + m't^n)\frac{\sigma}{E_s}\left(\frac{l}{b}\right)^4 \tag{13}$$

Noting, as before, that the relative density of the foam, ρ^*/ρ_s, is proportional to $(b/l)^2$ and setting $E^* = C_1(\rho^*/\rho_s)^2 E_s$ (Equation 7) we obtain

$$\varepsilon(\sigma, t) \propto (\varepsilon_0' + m't^n)\frac{\sigma}{C_1(\rho^*/\rho_s)^2 E_s} \tag{14}$$

$$= (\varepsilon_0' + m't^n)\frac{\sigma}{E^*} \tag{15}$$

giving the strain in the foam in terms of the creep parameters of the solid cell-wall material (ε_0, m, n), the initial Young's modulus of the foam, E^*, and the stress on the foam, σ. Because all of the constants of proportionality are the same in the elastic and creep analyses, the constant C_1 is the same in both cases.

In terms of creep compliance of the foam, $J^*(t)$, we can write

$$J^*(t) = \frac{\varepsilon}{\sigma} = \frac{(\varepsilon_0' + m't^n)}{E^*} \tag{16}$$

Noting that the creep compliance of the solid, $J_s(t)$, is (Equation 9)

$$J_s(t) = \frac{\varepsilon_s}{\sigma_s} = \frac{\varepsilon_0' + m't^n}{E_s}$$

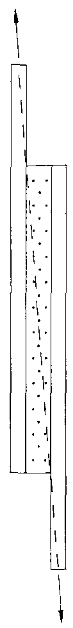

Figure 2 Loading jig for testing foam specimens in shear in the Instron testing machine.

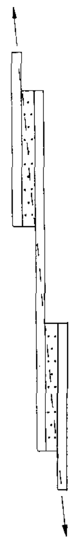

Figure 3 Loading jig for creep tests on foam specimens in shear.

gives

$$J^*(t) = J_s(t)\frac{E_s}{E^*} \tag{17}$$

For creep under shear loading this becomes

$$\gamma = \frac{(\gamma_0' + m't^n)\tau}{G^*}$$

and

$$J^*(t) = J_s(t)\frac{G_s}{G^*} \tag{18}$$

4. Experimental procedure

The experimental programme was aimed at measuring the creep response of several densities of rigid polyurethane foam in shear, as this is the primary loading on cores of structural sandwich panels. The experiments were designed to test the model of the previous section.

Four densities of rigid polyurethane foam were used in the experimental programme: 32, 48, 64 and 96 $\mathrm{kg\,m^{-3}}$. Owing to the rise of the gas during the foaming process, foams are generally axisymmetric in their structure and properties; the degree of anisotropy can be characterized by measuring the cell length and the Young's modulus in each of the three principal directions. The cell dimensions were found by measuring the mean intercept lengths in the principal directions on scanning electron micrographs. The Young's moduli in the three principal directions were obtained from compression tests, performed on 38 mm cubes of each density of foam.

The shear modulus and shear strength of each density of foam were measured in the most isotropic plane at a strain rate of roughly $5 \times 10^{-3}\,\mathrm{sec^{-1}}$ using the loading jig shown in Fig. 2; this jig is similar to that recommended in ASTM C273 [20] for shear testing of sandwich panel cores.

Creep tests were performed on the foam specimens by loading them in shear with dead weights as shown in Fig. 3. Four load levels were used for each density: roughly 10%, 20%, 30% and 40% of the shear strength. The highest stress level was limited to 40% of the shear strength as it is unlikely that foam cores in sandwich panels will be loaded beyond this level. One specimen was tested at each load level for each density of foam. LVDTs with a range of 1.27 mm, accurate to $\pm$ 0.0021 mm were mounted on to the loading jigs to measure deflections. The outputs from the 16 LVDTs were recorded using a digital data acquisition system. The specimens were loaded for 1200 h and then unloaded; deflection measurements were taken up to 450 h after unloading. The creep tests were done in a chamber at a constant temperature of 23 $\pm$ 1 °C and a constant relative humidity of 20% $\pm$ 2%.

5. Results and discussion

The mean intercept lengths of the cells and the Young's moduli in the three principal directions are listed in Tables I and II, respectively. For an axisymmetric foam with the rise in the x_1 direction we expect that $R_{23} \sim 1$ and that R_{12} and R_{13} are roughly equal. Similarly, we expect the Young's modulus in the x_1 direction to be larger than the roughly equal moduli in the x_2 and x_3 directions [21]. The data suggest that

TABLE I Shape anisotropy ratios

	Foam density ($\mathrm{kg\,m^{-3}}$)			
	32	48	64	96
l_1	0.277	0.223	0.223	0.181
l_2	0.263	0.199	0.212	0.147
l_3	0.217	0.189	0.192	0.150
R_{12}	1.056	1.122	1.050	1.230
R_{13}	1.280	1.181	1.164	1.203
R_{23}	1.212	1.053	1.109	0.979

Notes: l_i is the mean intercept length in the i direction; $R_{ij} = l_i/l_j$ = shape anisotropy ratio for the ij plane, x_1 is the rise direction.

TABLE II Young's modulus

	Foam density (kg m^{-3})			
	32	48	64	96
E_1^* (MPa)	5.48	11.1	17.2	36.7
E_2^* (MPa)	4.55	9.35	14.9	21.8
E_3^* (MPa)	3.46	7.26	10.2	26.2

Note: x_1 is the rise direction.

TABLE III Shear modulus and strength

	Foam density (kg m^{-3})			
	32	48	64	96
Isotropic plane	$x_1 - x_2$	$x_2 - x_3$	$x_1 - x_2$	$x_2 - x_3$
Shear modulus, G (MPa)	2.14	2.90	6.97	7.58
Shear strength, τ_{pl}^* (kPa)	113	157	299	346

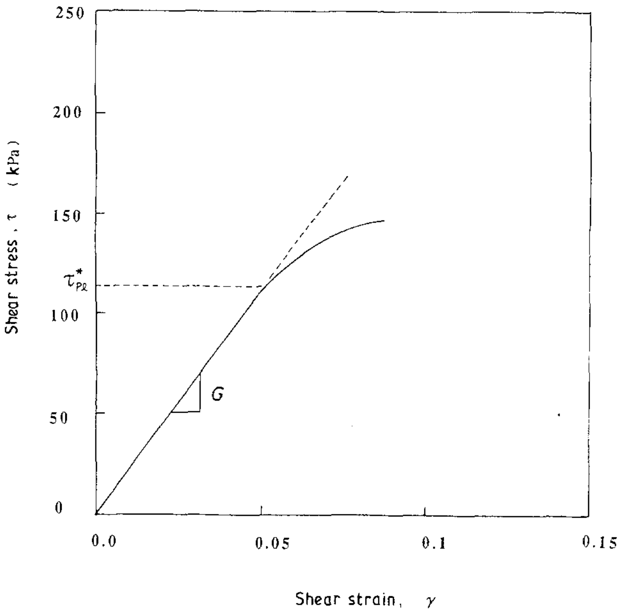

Figure 4 A typical stress–strain curve from a static shear test on a rigid polyurethane foam ($\rho = 32$ kg m^{-3}).

the foams are not axisymmetric; rather that they are orthotropic, with different mean intercept lengths and moduli in all three principal directions. To reduce the effect of the anisotropy in the creep tests the shear stress was applied in the most isotropic plane; this is the plane indicated at the top of Table III.

A typical stress–strain curve from one of the static shear tests is shown in Fig. 4. The shear modulus is given by the initial slope of the curve while the shear strength is defined as the first deviation from linearity; the moduli and strengths are listed in Table III. The shear stresses applied to the creep specimens are roughly 10%, 20%, 30% and 40% of the shear strengths given in Table III; exact values are given in Table IV, along with the initial shear modulus of each of the creep specimens at 5 sec loading.

The creep test results are shown in Figs 5 to 8. The shear strain data are plotted against time for each

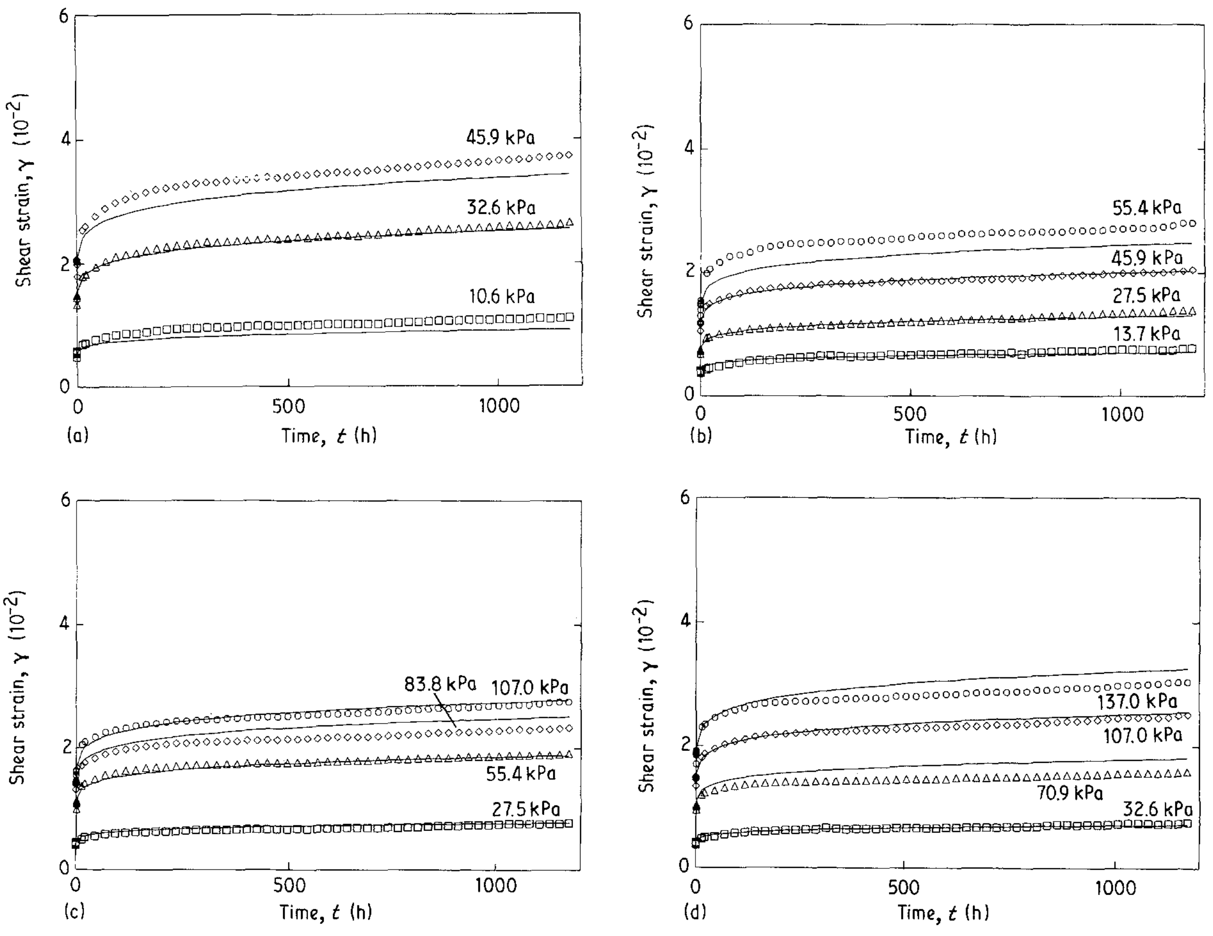

Figure 5 Creep shear strain plotted against time for four densities of foam: (a) 32 kg m^{-3} (b) 48 kg m^{-3} (c) 64 kg m^{-3} and (d) 96 kg m^{-3}. (———) Theoretical results.

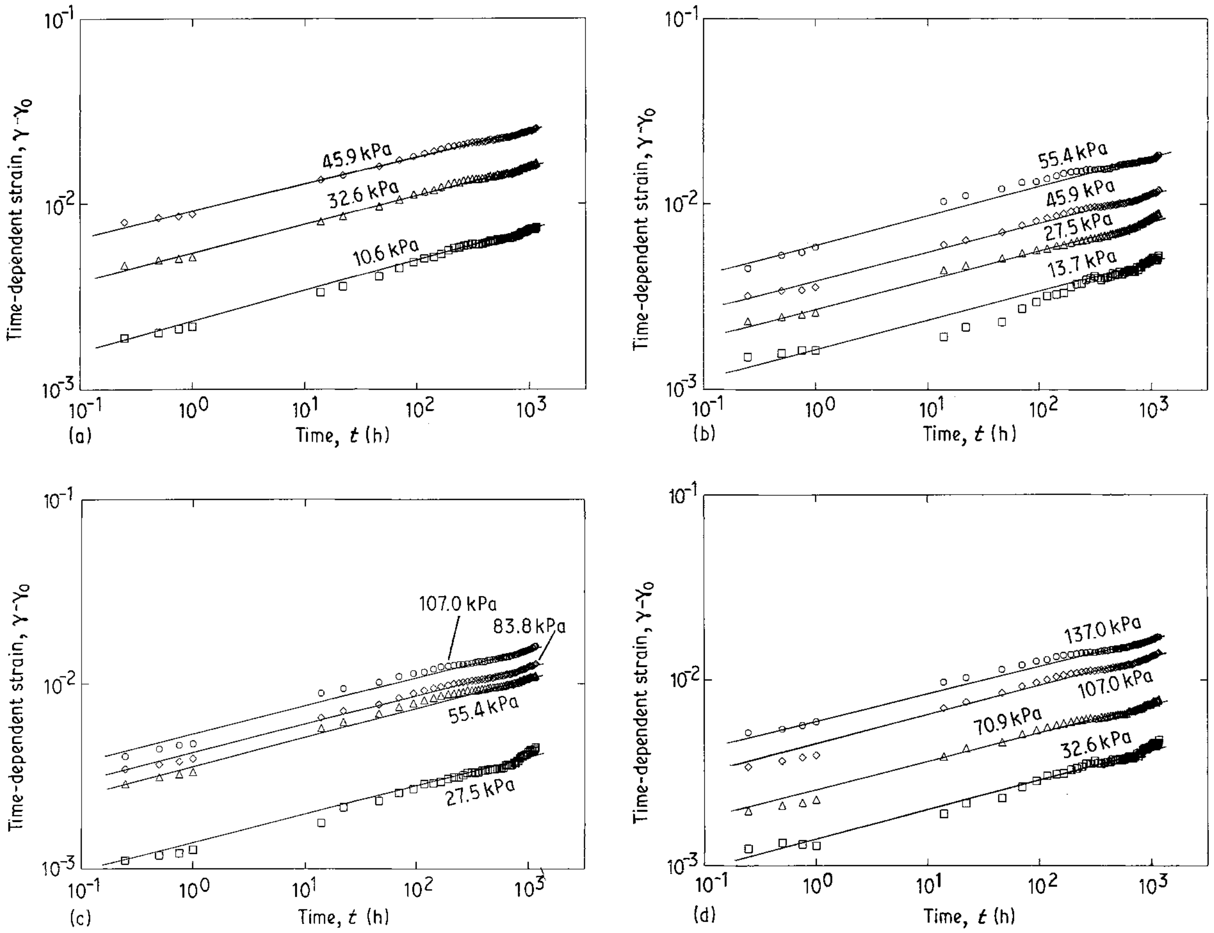

Figure 6 A double logarithmic plot of creep shear strain against time: (a) 32 $kg\,m^{-3}$, (b) 48 $kg\,m^{-3}$, (c) 64 $kg\,m^{-3}$, and (d) 96 $kg\,m^{-3}$. The data lie on a straight line indicating a power law dependence of creep strain on time.

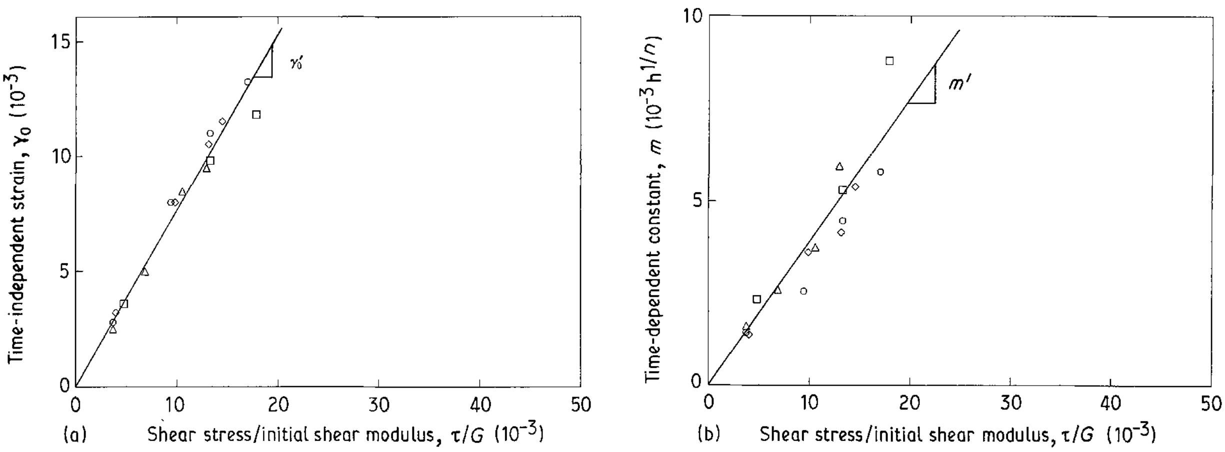

Figure 7 Measured values of (a) γ_0 and (b) m_0 plotted against shear stress normalized by the shear modulus for each creep specimen (Table IV). Both γ_0 and m_0 are linear functions of stress. (□) 32 $kg\,m^{-3}$, (△) 48 $kg\,m^{-3}$, (◇) 64 $kg\,m^{-3}$, (○) 96 $kg\,m^{-3}$.

density of foam and each stress level in Fig. 5. The data are reduced to give plots of $\log(\gamma - \gamma_0)$ against log (time) in Fig. 6; values for γ_0 were found using a trial and error procedure such that $\log(\gamma - \gamma_0)$ was a linear function of log (time), that is

$$\log(\gamma - \gamma_0) = \log(m) + n\log(t) \quad (19)$$

or

$$\gamma = \gamma_0 + mt^n \quad (20)$$

The value of m is given by the intercept of the plot at a time of 1 h, that of n by the slope; the results are summarized in Table V. The value of n was found to be constant at 0.155, independent of both stress and foam density; this is identical with that found by Findley and his co-workers for both solid and foamed polyurethane [14, 15]. γ_0 and m are both linear functions of stress normalized by the shear modulus (Fig. 7) giving

$$\gamma = \gamma_0 + mt^n = \frac{(\gamma'_0 + m't^n)\tau}{G^*} \quad (21)$$

with $\gamma'_0 = 0.761$ ($R^2 = 0.99$) and $m' = 0.384\ h^{1/n}$ ($R^2 = 0.97$); both constants are found by linear regression. In terms of the creep compliances of the foam

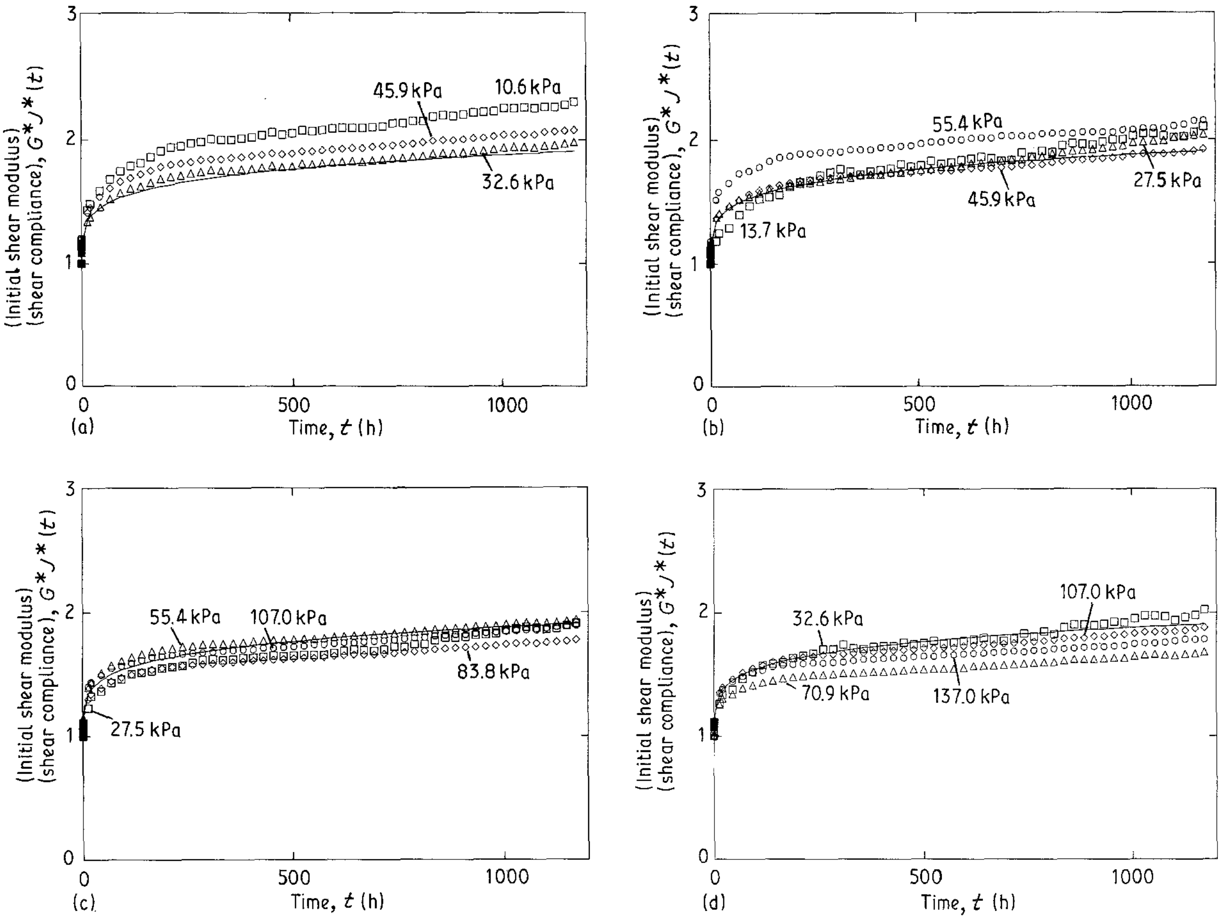

Figure 8 The product of creep compliance times shear modulus plotted against time: (a) 32 kg m^{-3} (b) 48 kg m^{-3} (c) 64 kg m^{-3} and (d) 96 kg m^{-3}. (———) Theoretical results.

and the solid we can write

$$J^*(t)G^* = \gamma_0' + m't^n$$
$$= J_s(t)G_s \qquad (22)$$

The product of the creep compliance and the shear modulus of each foam specimen is a constant function of time, independent of the stress level and the foam density; note that this is in agreement with Nolte and Findley's data for solid and foamed polyurethane. Data for this product are plotted against time in Fig. 8, along with the prediction of Equation 21. The product tends to decrease slightly as the density of the foam increases; agreement is still reasonably good, however. The results of the model are also compared with the strain data in Fig. 5, in which the solid lines represent the model (Equation 18). Agreement is good; the model predicts the measured creep strains to within 10% in all cases.

The data for unloading are treated in a similar way as that for loading; they are fitted to an equation of the form of Equation 20 with the strain replaced by the recovered strain, $\gamma_{max} - \gamma$ and with time, t, replaced by

TABLE IV Creep specimens – stress levels and shear moduli

	Foam density (kg m^{-3})			
	32	48	64	96
τ_1 (kPa)	10.6	13.7	27.5	32.6
G_1^* (MPa)[a]	2.20	3.68	6.78	8.73
τ_2 (kPa)	[b]	27.5	55.4	70.9
G_2^* (MPa)[a]	[b]	4.00	5.61	7.50
τ_3 (kPa)	32.6	45.9	83.8	107
G_3^* (MPa)[a]	2.44	4.32	6.37	8.01
τ_4 (kPa)	45.9	55.4	107	137
G_4^* (MPa)[a]	2.56	4.26	7.34	8.06

[a] Shear moduli are calculated from the deflection at 5 sec during the creep test.
[b] The 32 kg m^{-3} specimen with τ_2 broke during loading.

TABLE V Data for creep parameters γ_0, m and n for constant load

Density (kg m^{-3})	τ (kPa)	γ_0 ($\times 10^{-3}$)	m (10^{-3} h$^{1/n}$)	n
32	10.6	3.6	2.34	0.155
	32.6	9.8	5.31	0.155
	45.9	11.8	8.79	0.155
48	13.7	2.5	1.61	0.155
	27.5	5.0	2.59	0.155
	45.9	8.5	3.76	0.155
	55.4	9.5	5.96	0.155
64	27.5	3.2	1.37	0.155
	55.4	8.0	3.6	0.155
	83.8	10.5	4.16	0.155
	107	11.5	5.39	0.155
96	32.6	2.8	1.43	0.155
	70.9	8.0	2.54	0.155
	107	11.0	4.47	0.155
	137	13.2	5.79	0.155

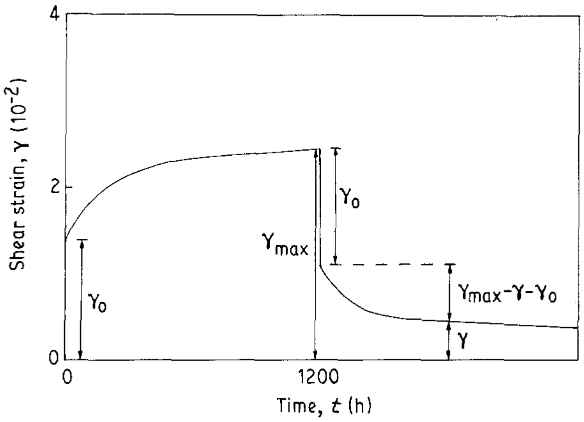

Figure 9 Schematic illustration of shear strain against time, showing the time independent strain, γ_0, the maximum strain, γ_{max}, the current strain, γ, and the recovered creep strain, $\gamma_{max} - \gamma_0 - \gamma$.

the time after unloading, t_u

$$\gamma_{max} - \gamma = \gamma_0 + m_u t_u^{n_u} \tag{23}$$

where γ_{max} is the strain at the time of unloading (in this case at $t = 1200$ h), γ is the current strain and γ_0 is taken to be equal for both loading and unloading (Fig. 9). The shear strain data for unloading are first plotted against time in Fig. 10; a double log plot of the recovered creep strain, $\gamma_{max} - \gamma_0 - \gamma$, against time after unloading, t_u, is then made (Fig. 11), from which values for m_u and n_u are found (Table VI). Because n_u is a constant and m_u is a linear function of stress

TABLE VI Data for the creep parameters γ_{max}, m_u, and n_u for unloading

Density ($kg\,m^{-3}$)	τ (kPa)	γ_{max} ($\times 10^{-3}$)	m_u ($10^{-3}\,h^{1/n}$)	n_u
32	10.6	11.2	1.59	0.105
	32.6	26.5	3.90	0.105
	45.9	37.3	6.86	0.105
48	13.7	7.98	1.50	0.105
	27.5	14.0	2.00	0.105
	45.9	20.5	2.92	0.105
	55.4	28.2	5.14	0.105
64	27.5	7.82	0.85	0.105
	55.4	–	–	–
	83.8	23.6	3.31	0.105
	107	27.5	4.17	0.105
96	32.6	7.66	1.36	0.105
	70.9	16.0	2.00	0.105
	107	–	–	–
	137	30.4	4.89	0.105

Note: The deflection gauges on two specimens broke during unloading.

(Fig. 12) the strain on unloading can be written as

$$\gamma = (\gamma'_0 + m'_u t_u^{n_u}) \frac{\tau}{G^*} \tag{24}$$

with $\gamma'_0 = 0.761$ ($R^2 = 0.99$), $m'_u = 0.310$ ($R^2 = 0.96$) and $n_u = 0.105$. The solid lines in Fig. 10 are plots of Equation 23; they give a good description of the data

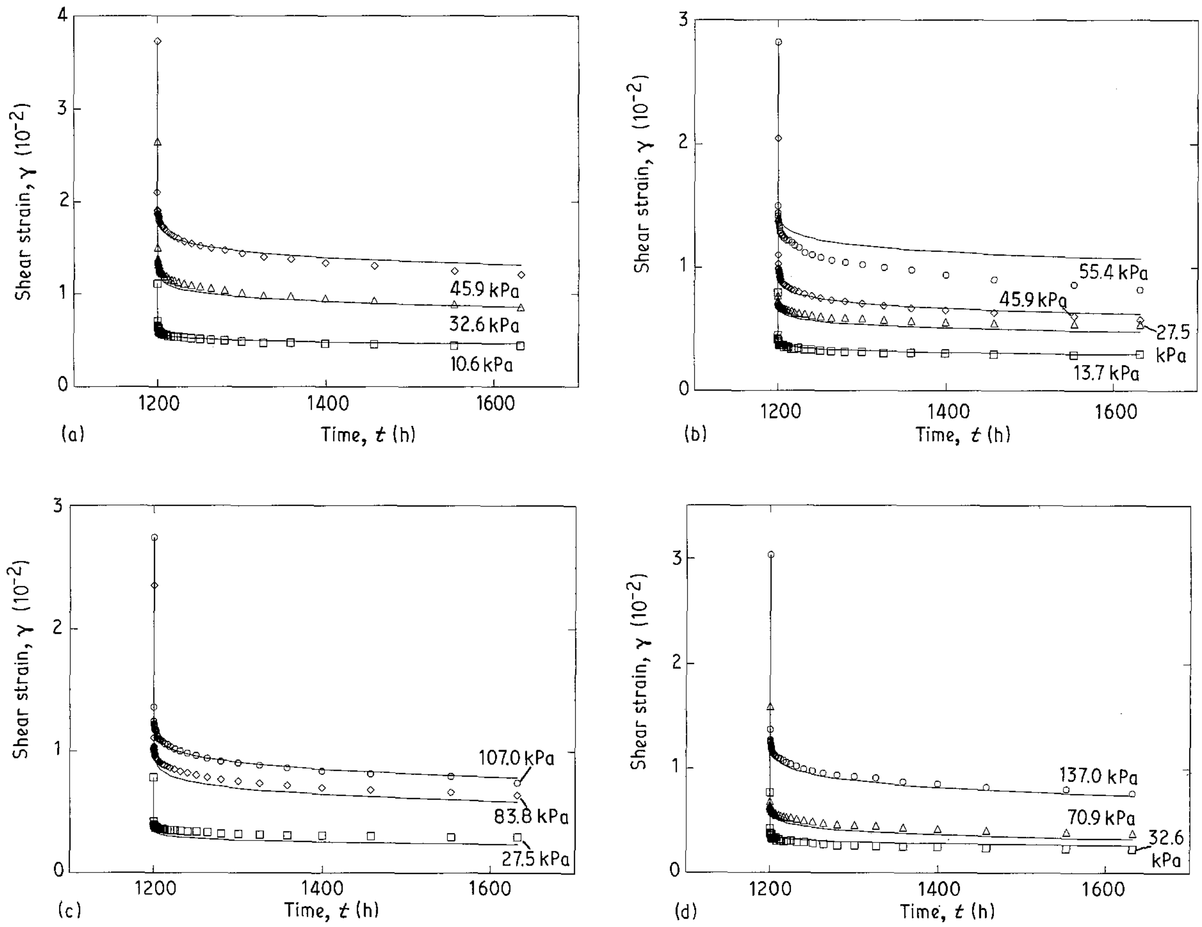

Figure 10 Shear strain plotted against time after unloading: (a) 32 $kg\,m^{-3}$ (b) 48 $kg\,m^{-3}$ (c) 64 $kg\,m^{-3}$ and (d) 96 $kg\,m^{-3}$. (———) Theoretical results.

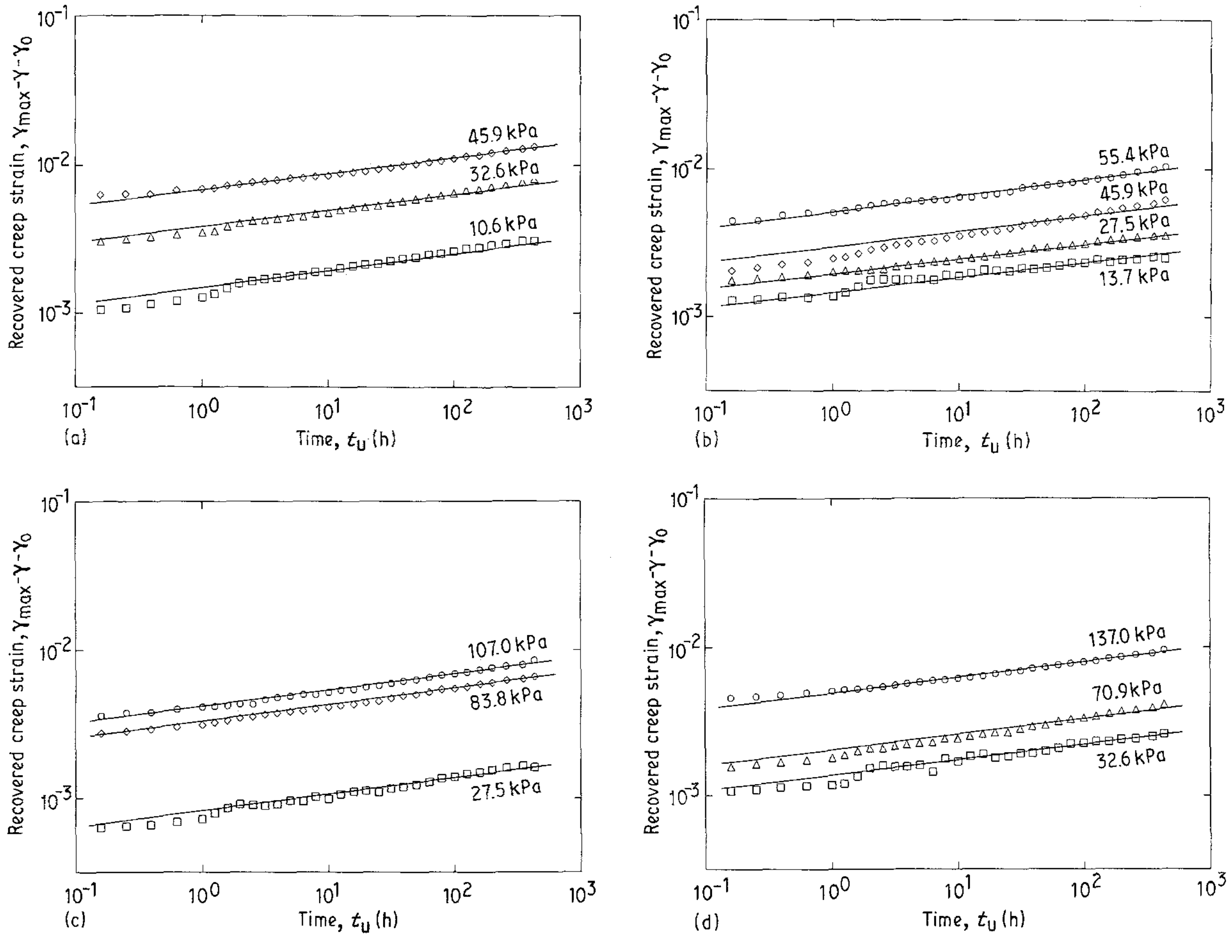

Figure 11 A double logarithmic plot of recovered creep shear strain against time after unloading: (a) 32 kg m^{-3}, (b) 48 kg m^{-3}, (c) 64 kg m^{-3} and (d) 96 kg m^{-3}. The data lie on a straight line indicating a power law dependence of recovered creep shear strain on time after unloading.

in all cases except the highest load level on the 48 kg m^{-3} density foam.

6. Conclusions

The linear and non-linear viscoelastic behaviour of polymer foams have been analysed, giving expressions for the creep of a foam in terms of the creep of the solid polymer and the relative density of the foam. A series of creep tests on rigid polyurethane foams of different densities suggest that at stress levels less than half the yield strength of the foam:

(a) rigid polyurethane foam is linear viscoelastic, even at relatively long loading times;

(b) the creep strain for loading is well described by Equation 21

$$\gamma = \frac{(\gamma'_0 + m't^n)\tau}{G^*}$$

with $\gamma'_0 = 0.761$, $m' = 0.384\ \mathrm{h}^{1/n}$ and $n = 0.155$;

(c) the creep strain for unloading is well described by Equation 24

$$\gamma = (\gamma'_0 + m'_u t_u^{n_u})\frac{\tau}{G^*}$$

with $\gamma'_0 = 0.761$, $m'_u = 0.310\ \mathrm{h}^{1/n_u}$ and $n_u = 0.105$;

(d) creep of any density of a rigid polyurethane foam at stress levels below half the shear strength can be found knowing the creep compliance of a single density and the shear moduli of both densities, as described by the result of the model for linear viscoelastic foams (Equation 22)

$$J^*(t)G^* = J_s(t)G_s$$

These results, combined with Just's observation [17] that long-term (10 y) creep of rigid polyurethane foam can be estimated on the basis of 1000 h tests, give a satisfactory means of estimating long-term creep for any density of a linear viscoelastic foam.

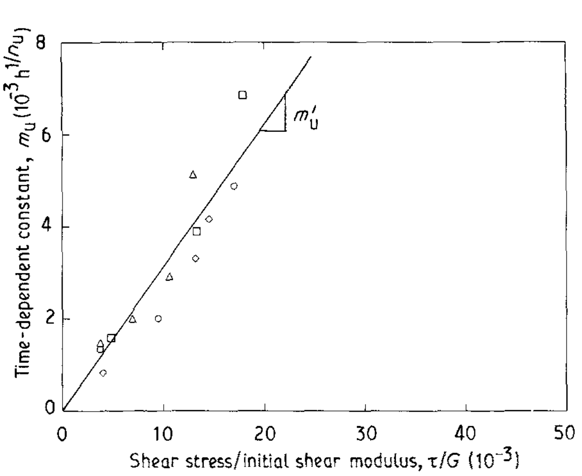

Figure 12 Measured values of m_u plotted against shear stress normalized by the shear modulus for each creep specimen (Table IV); m_u is a linear function of stress. (□) 32 kg m^{-3}, (△) 48 kg m^{-3}, (◇) 64 kg m^{-3}, (○) 96 kg m^{-3}.

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Appendix: creep of a non-linear viscoelastic foam

The creep of a foam made from a non-linear viscoelastic solid is found in the same way as indicated in Section 3; the calculation is complicated by the analysis of the beam deflection of a non-linear solid. We assume that plane cross-sections of the beam remain plane, and that the solid is isotropic and creeps in both tension and compression according to (Equation 5a)

$$\begin{aligned}\varepsilon_s &= \varepsilon_0 + mt^n \\ &= (\varepsilon_0' + m't^n)\sinh\left(\frac{\sigma_s}{\sigma_0}\right) \qquad \text{(A1)}\end{aligned}$$

Consider first the bending of a solid square beam, of section b^2, which creeps according to Equation A1. Following the method of Findley and Poczatek [22], we find that the strain in the beam is equal to

$$\varepsilon_s = \frac{y}{r} \qquad \text{(A2)}$$

where y is the distance from the neutral axis and r is the radius of curvature of the beam. Equating A1 and A2 and inverting gives

$$\sigma_s = \sigma_0 \sinh^{-1}\left(\frac{y}{r(\varepsilon_0' + m't^n)}\right) \qquad \text{(A3)}$$

or

$$\sigma_s = \sigma_0 \sinh^{-1}(Ny) \qquad \text{(A4)}$$

where

$$N = \frac{1}{r(\varepsilon_0' + m't^n)} \qquad \text{(A5)}$$

Using the conditions of equilibrium, Findley and Poczatek [22] show that the neutral axis is at the centre of the beam and that the moment, M, at a section in the beam is given by

$$\begin{aligned}M &= b\int_{-b/2}^{b/2} \sigma y\,\mathrm{d}y \\ &= b\sigma_0 \int_{-b/2}^{b/2} y \sinh^{-1}(Ny)\mathrm{d}y \qquad \text{(A6)}\end{aligned}$$

After integrating and rearranging, they find

$$\begin{aligned}\frac{8M}{\sigma_0 b^3} &= \left(\frac{Nb}{2}\right)^{-2}\left\{\left[2\left(\frac{Nb}{2}\right)^2 + 1\right]\sinh^{-1}\left(\frac{Nb}{2}\right)\right. \\ &\quad \left. - \left(\frac{Nb}{2}\right)\left[1 + \left(\frac{Nb}{2}\right)^2\right]^{1/2}\right\} \qquad \text{(A7)}\end{aligned}$$

Because N depends only on the bending moment, M, the size of the beam, b, and the beam material property, σ_0, and is independent of time, the stress distribution within the beam is also independent of time (Equation A3). To solve for the stress distribution in terms of the bending moment in Equation A3, N must be solved in terms of M from Equation A7. Because there is no closed form solution of Equation A7 we approximate the function as

$$\frac{Nb}{2} = \sinh\left(\frac{8C_1 M}{\sigma_0 b^3}\right) \qquad \text{(A8)}$$

With $C_1 = 0.65$, the maximum error arising from using Equation A8 rather than Equation A7 is about 10% for values of $Nb/2$ up to 5.

Consider next the creep of a foam made up of a non-linear viscoelastic material obeying Equation A1. Under a uniaxial stress, σ^*, the cell walls bend, as shown schematically in Fig. 1b. The bending deflection, δ, is proportional to l^2/r or, using Equation A5,

$$\delta \propto l^2 N(\varepsilon_0' + m't^n) \qquad \text{(A9)}$$

Substituting Equation A8 for N, setting the strain in the foam, ε, proportional to δ/l and setting the bending moment on the cell wall, M, proportional to σl^3 gives the strain in the foam, ε, as a function of the stress on the foam, σ, and the time, t

$$\varepsilon \propto \frac{l}{b}(\varepsilon_0' + m't^n)\sinh\left[C_1 \frac{\sigma}{\sigma_0}\left(\frac{l}{b}\right)^3\right] \qquad \text{(A10)}$$

Noting that the relative density, ρ^*/ρ_s, is proportional to $(b/l)^2$ we obtain

$$\varepsilon \propto \left(\frac{\rho_s}{\rho^*}\right)^{1/2}(\varepsilon_0' + m't^n)\sinh\left[C_1 \frac{\sigma}{\sigma_0}\left(\frac{\rho_s}{\rho^*}\right)^{3/2}\right] \qquad \text{(A11)}$$

Note that when the argument of the hyperbolic sine is large (corresponding to $Nb/2 > 5$) the sinh factor is replaced by an exponential. For the special case of a small argument, the result reduces to the linear viscoelastic case Equation 15.

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